A Short Introduction to Directed Acyclic Graphs (DAGs)

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Karolinska Institutet

Background - Counterfactuals

Counterfactuals

Y_i^x denotes the potential outcome Y of individual i under treatment x.

E.g. my (i) sleep quality tonight (Y) if I ate pasta ($x = 1$) instead of oats ($x = 0$) for dinner.

Problem

- We are interest in $\mathbb{E}[Y^{x=0}] - \mathbb{E}[Y^{x=1}]$
- We know $\mathbb{E}[Y|x = 0] - \mathbb{E}[Y|x = 1]$

Hence, we need a method for translating our contrafactual outcomes of interest into observable quantities.

Solution: Pearl's *do*-calculus, which requires 3 assumptions.

Background - Identifiability Assumption

1. Consistency ($Y_i^x = Y_i$, if $X_i = x$)

The counterfactual outcome Y_i^x corresponds to the observed outcome Y_i if individual i received treatment x in the real world.

Background - Identifiability Assumption

1. Consistency ($Y_i^x = Y_i$, if $X_i = x$)

2. Conditional Exchangeability ($Y^x \perp\!\!\!\perp X | L$)

The counterfactual outcome is independent of the observed treatment given some adjustment set L.

Background - Identifiability Assumption

1. Consistency ($Y_i^x = Y_i$, if $X_i = x$)
2. Conditional Exchangeability ($Y^x \perp\!\!\!\perp X|L$)

3. Positivity ($\mathbb{P}[X = x, L = l] > 0$)

It should in theory be possible to identify both treated and untreated individuals for each possible combination of the variables included in the adjustment set L.

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Idetifiability Assumptions and DAGs

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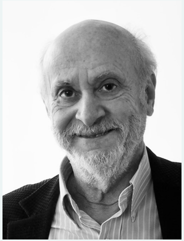
How to asses exchangeability?

Judea Pearl

vs.



Donald B. Rubin

The Basic Ingridiens of DAGs

Mathematically speaking, a DAG is a visual representation of a joint distribution of variables defined by:

- **Nodes**: Variables in our causal network
- **Arrows**: Direction of causation
- Note that the absence of an arrow is a stronger assumption than the presence of it, i.e., complete independence.

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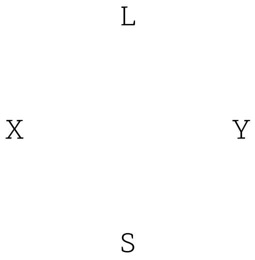

Figure 1. Some nodes

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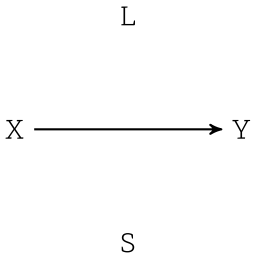

Figure 1. A direct effect

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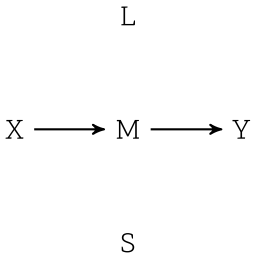

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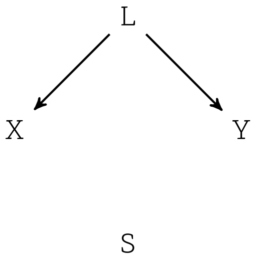

Figure 1. A confounder

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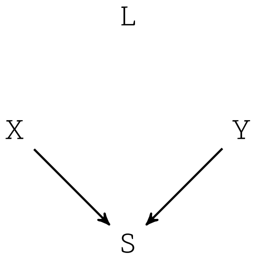

Figure 1. A collider

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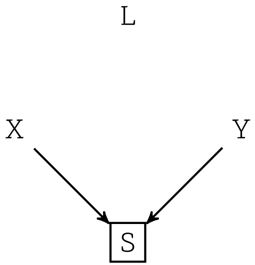

Figure 1. Conditioned on a collider

From Counterfactuals to Reality via *do*-calculus

Back-door criteria

We know as a results from Pearl's *do*-calculus that exchangability holds if there is no open path between X and Y in a DAG in which all outgoing arrows from X are removed.

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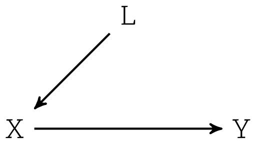

Figure 2. The closed back door

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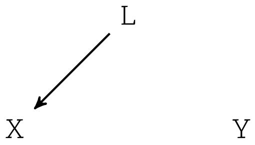

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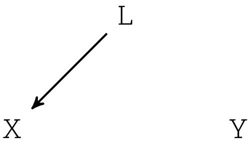

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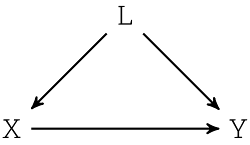

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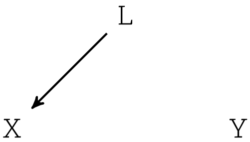

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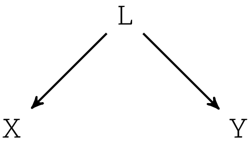

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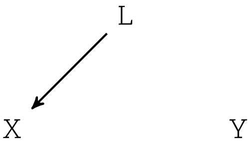

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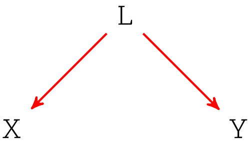

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A Randomised Experiment

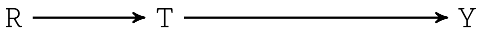

Figure 4. DAG of a randomised experiment

- R: Randomisation
- T: Treatment
- Y: Outcome (weight loss)
- P: Prognostic factors (baseline BMI)
- S: Collider (fatigue)
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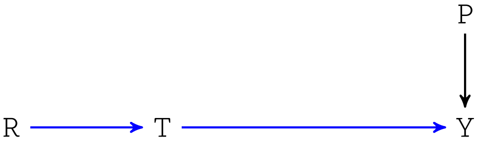

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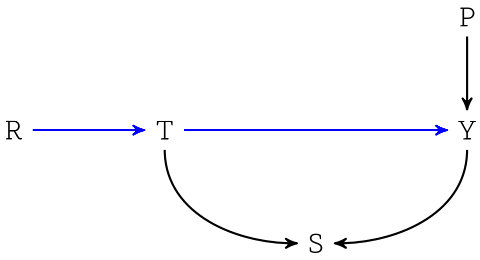

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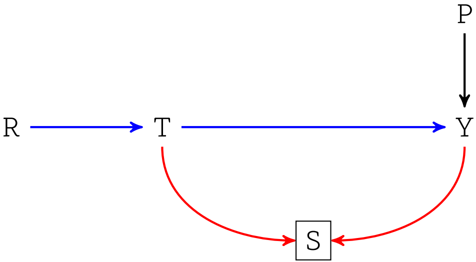

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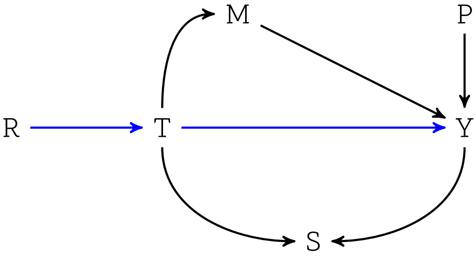

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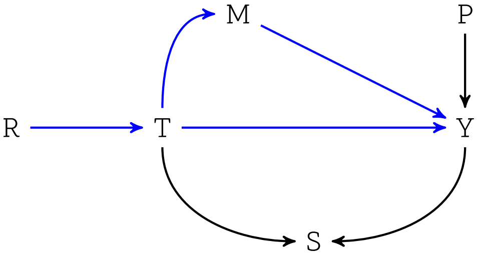

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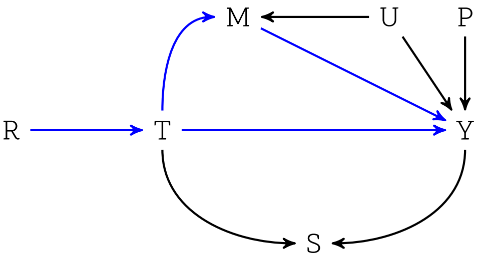

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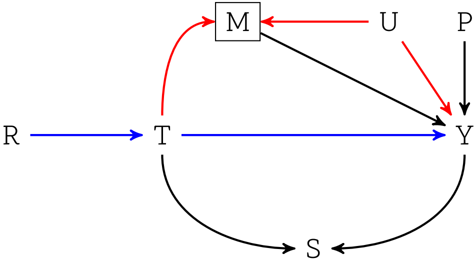

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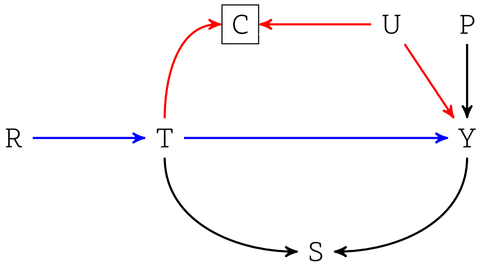

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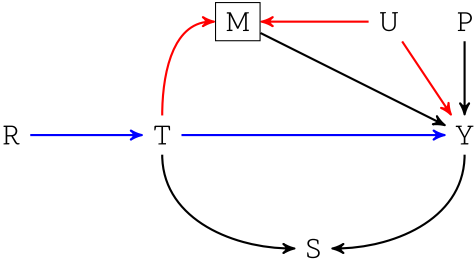

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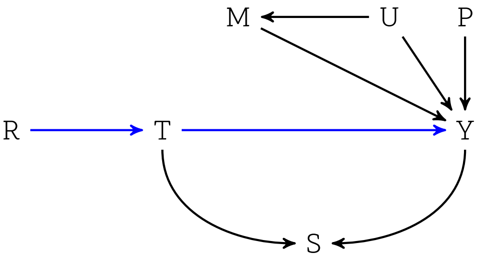
A Blinded Randomised Experiment



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A Blinded Randomised Experiment with ICEs

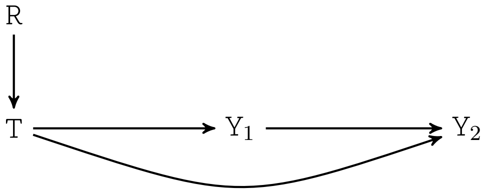

Figure 5. DAG of an ICE mechanism

- R: Randomisation
- T: Treatment
- C: ICE (treatment discontinuation)
- Y_t: Outcome at time t
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- M_{y2}: A mechanism for assigning a value to Y.

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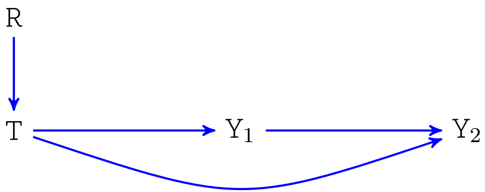

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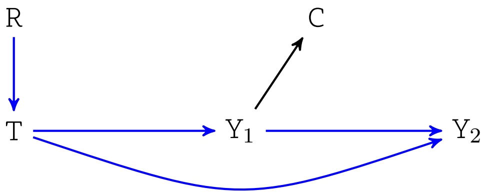

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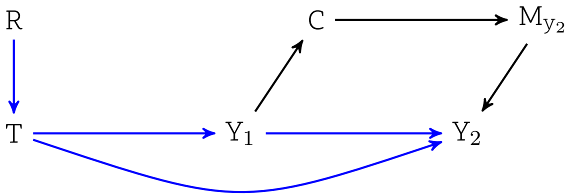

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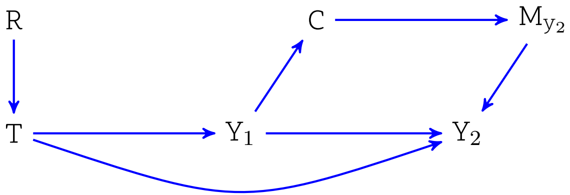

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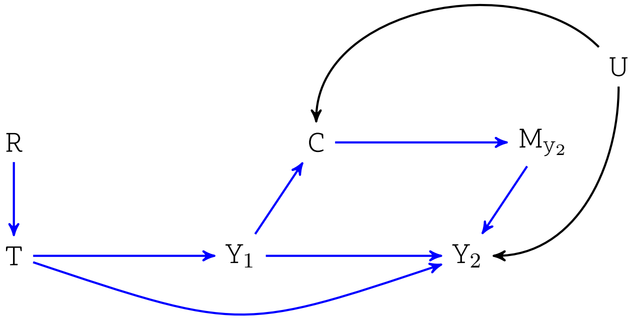

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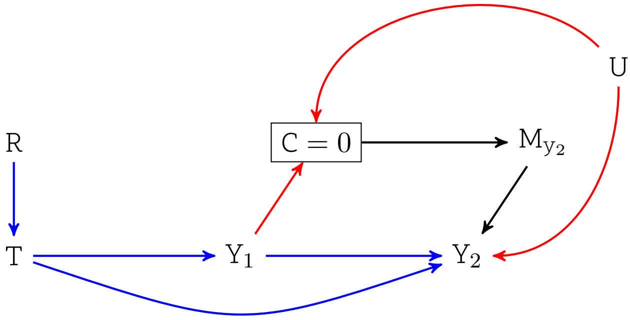

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A Blinded Randomised Experiment with Landmark Analysis

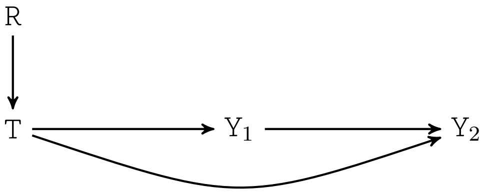

Figure 6. DAG of a landmark analysis in a RCT

- R: Randomisation
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- $\mathrm{Y_t}$: Outcome at time t (survival)
- U: Confounder (patient global health)

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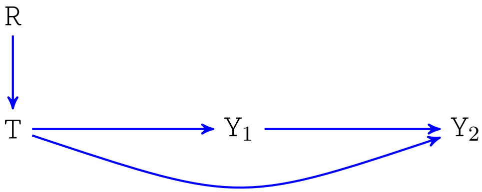

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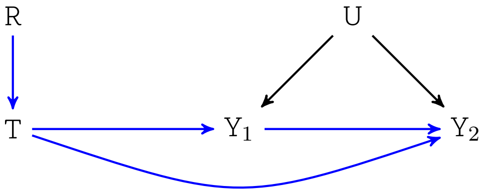

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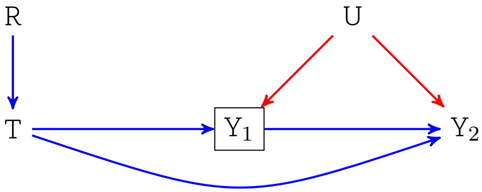

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Contact Information

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Link to the slides and materials